Università di Bologna – Campus di Rimini – Corso di laurea in Farmacia e CQPS

Esame MATEMATICA 14/09/2015 – Docente: Stefano Bordoni

STUDENTE: __________________________; CORSO di LAUREA: ____________

MATRICOLA: ________; N° documento: _____________; FIRMA: ____________

1. Calcolare: $\log_k(k \cdot \sqrt[2]{k})$, $\dfrac{(n+1)! - n!}{(n-1)!}$ [2]

2. Risolvere:

$\left(\frac{1}{5}\right)^{-x+1} \geq \frac{1}{25}$; $\left|7x - 3x^2\right| \geq 0$; $\sqrt[2]{3 - x^2} \geq -1$; $\sqrt[3]{6 - x^2} < -1$; $\log_{\frac{3}{4}}(x^2 - \frac{2}{3}) > -1$. [7]

3. Calcolare la funzione derivata e una funzione primitiva della funzione $y = \dfrac{1}{\sqrt[2]{x}}$ [3]

4. Determinare dominio, grafico e codominio (studio globale) della funzione

$$y = f(x) = \ln(x+1) .$$

Determinare la funzione *y = g(x)* simmetrica di *f(x)* rispetto l'asse *y*, e la funzione derivata $f'(x)$. [6]

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5. Eseguire lo studio analitico completo della funzione $y = x \cdot \ln(x)$. [7]

6. Dopo aver determinato un intervallo sul quale la funzione $y = f(x) = x^3 - 1$ è invertibile, determinare la funzione inversa, inclusi dominio, grafico e codominio. [3]

7. In una moneta non truccata, qual è la probabilità che escano 4 teste su 4 lanci? [2]

<u>PER LA LODE</u>: Risolvere graficamente la disequazione $\ln(|x|) < |x|$.

VOTO: _______

Rimini Campus - C.L. Farmacia e CQPS

Matematica 14/09/2015

1) • $K^z = K \cdot \sqrt[2]{K}$ $\quad K^z = K \cdot K^{1/2}$ $\quad K^z = K^{3/2}$ $\quad z = \frac{3}{2} \rightarrow \log_K (K \cdot \sqrt[2]{K}) = \frac{3}{2}$

• $\dfrac{(n+1)\cdot n(n-1)! - n\cdot(n-1)!}{(n-1)!} = \dfrac{\cancel{(n-1)!}\,[(n+1)\cdot n - n]}{\cancel{(n-1)!}} = n^2+n-n = n^2$

2) • $\left(\frac{1}{5}\right)^{-x+1} \geq \left(\frac{1}{5}\right)^2 \rightarrow \quad -x+1 \leq 2 \quad -x \leq +1 \quad x \geq -1$ $\quad$ (base<1)

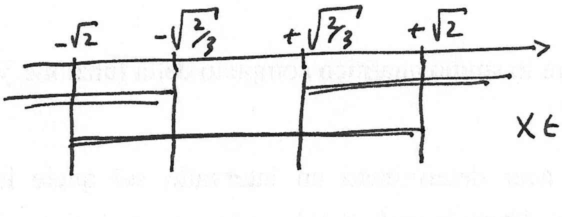
oppure $5^{x-1} \geq 5^{-2} \rightarrow \quad x-1 \geq -2 \quad x \geq -1$ $\quad$ (base>1)

• $|7x - 3x^2| \geq \phi$ $\quad \forall x \in \mathbb{R}$ per definizione di valore assoluto

• $\sqrt[2]{3-x^2} \geq -1$ $\quad 3-x^2 \geq \phi \quad x^2-3 \leq \phi \quad x \in [-\sqrt{3}; +\sqrt{3}]$

• $\log_{3/4}\left(x^2 - \frac{2}{3}\right) > -1$ $\quad \begin{cases} x^2 - \frac{2}{3} > \phi \\ x^2 - \frac{2}{3} < \left(\frac{3}{4}\right)^{-1} \end{cases}$ $\quad \begin{cases} x \in \left]-\infty; -\sqrt{\frac{2}{3}}\right[\cup \left]+\sqrt{\frac{2}{3}}; +\infty\right[\\ x^2 - \frac{2}{3} < \frac{4}{3}; \; x^2 - \frac{6}{3} < \phi; \; x^2 - 2 < \phi \end{cases}$

$\begin{cases} / \\ x \in \left]-\sqrt{2}; +\sqrt{2}\right[\end{cases}$



$x \in \left]-\sqrt{2}; -\sqrt{\frac{2}{3}}\right[\cup \left]+\sqrt{\frac{2}{3}}; +\sqrt{2}\right[$

3) $y = \dfrac{1}{\sqrt[2]{x}} = \dfrac{1}{x^{1/2}} = x^{-1/2}$ $\quad x \in \mathbb{R}^+$

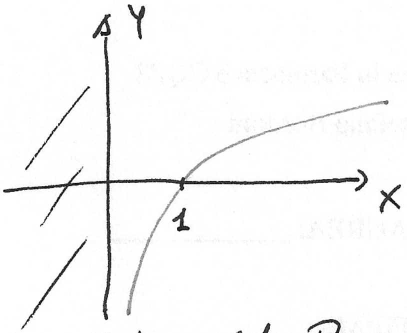
Derivate: $y' = -\frac{1}{2} \cdot x^{-\frac{1}{2}-1} = -\frac{1}{2} x^{-3/2} = -\frac{1}{2} \frac{1}{x^{3/2}} = -\frac{1}{2} \frac{1}{\sqrt[2]{x^3}} = \dfrac{-1}{2x \cdot \sqrt[2]{x}}$

Primitive: $\phi = \dfrac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \dfrac{x^{1/2}}{\frac{1}{2}} = 2 \cdot \sqrt[2]{x}$

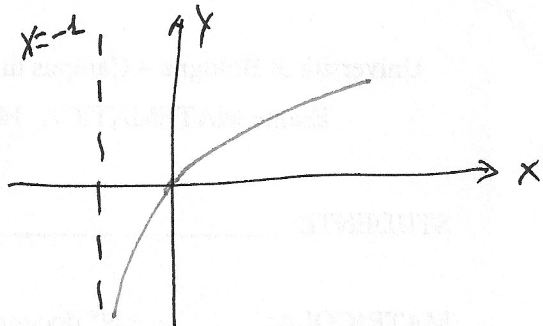
• $\sqrt[3]{6-x^2} < -1$ $\quad 6-x^2 < (-1)^3; \; 6-x^2 < -1; \; -x^2 < -7 \quad x^2 > 7$

$x \in \left]-\infty; -\sqrt{7}\right[\cup \left]+\sqrt{7}; +\infty\right[$

4) • $y = \ln(x)$ $\xrightarrow[1L]{T_x}$ $y = \ln(x+1)$



$D: \mathbb{R}^+$; $Cd: \mathbb{R}$



$D: x \in]-1; +\infty[$; $Cd: \mathbb{R}$

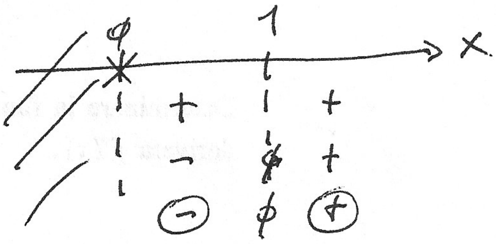
• $y = f(x) \xrightarrow{S_Y} y = f(-x)$: $y = \ln(-x+1)$

• $y = f'(x) = \dfrac{1}{x+1}$

5) $y = x \cdot \ln(x)$ $D: \mathbb{R}^+ \rightarrow \nexists$ invarianze per simmetria

• $?x: \; x \cdot \ln(x) \geq 0$ $\rightarrow ?\, x \geq 0$ $\quad x \geq 0$

$\rightarrow ?\, \ln(x) \geq 0$ $\quad x \geq 1$



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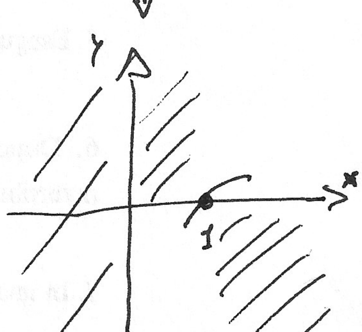


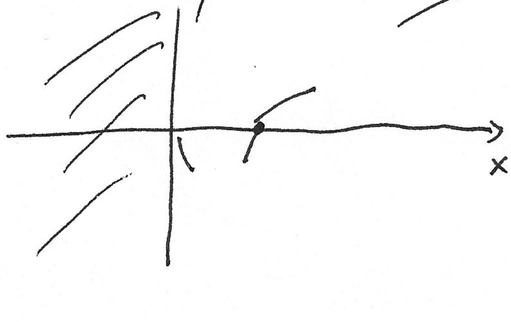
• $\lim\limits_{x \to 0^+} \left[x \cdot \ln(x)\right] = 0^+ \cdot \ln(0^+) = 0^+ \cdot (-\infty)$

INDETERMINATO

$\rightarrow \lim\limits_{x \to 0^+} \dfrac{\ln(x)}{1/x} \equiv \lim\limits_{x \to 0^+} \dfrac{1/x}{-1/x^2} = \lim\limits_{x \to 0^+} (-x) = 0^-$

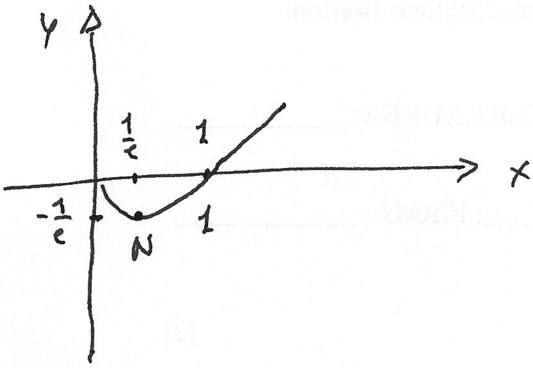
$\lim\limits_{x \to +\infty} x \cdot \ln(x) = +\infty \cdot (+\infty) = +\infty$

$\Downarrow$

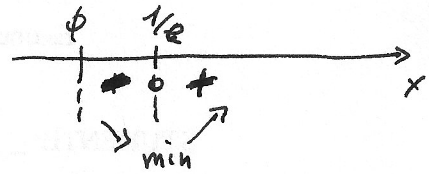


- $y' = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$

$?x: \quad \ln(x)+1 \geq \phi \qquad \ln(x) \geq -1 \qquad x \geq e^{-1} \qquad x \geq \frac{1}{e}$



$\Leftarrow$

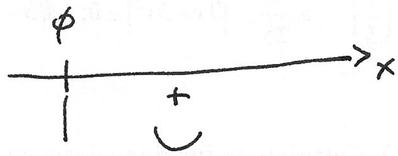


$y\left(\frac{1}{e}\right) = \frac{1}{e} \cdot \ln\left(\frac{1}{e}\right) = \frac{1}{e} \cdot (-1)$

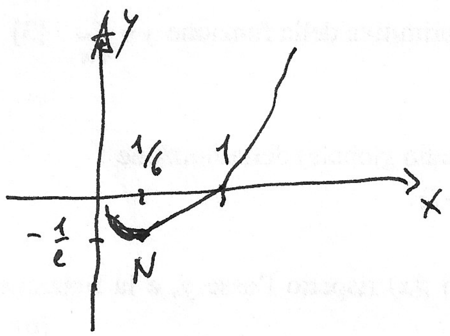
$= -\frac{1}{e}$

$N \equiv \left(\frac{1}{e}; -\frac{1}{e}\right)$

- $y'' = \frac{1}{x} \qquad ?x: \frac{1}{x} \geq \phi \qquad \underline{x > \phi}$



$\Rightarrow$

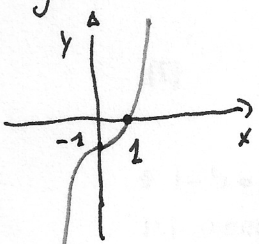


NON CI SONO PUNTI DI FLESSO

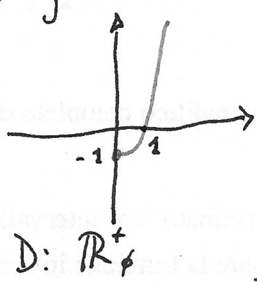
6) $y = x^3 - 1 \quad \longrightarrow$ INIETTIVA E QUINDI INVERTIBILE $\forall x \in \mathbb{R}^+_\phi$ $\quad \longrightarrow \quad y = x^3 - 1 \quad \xrightarrow{\text{INVERSA}} \quad x = y^3 - 1 \qquad y^3 = x + 1$

$\underline{y = \sqrt[3]{x+1}}$ $\quad D: [-1; +\infty[$

$Cd: \mathbb{R}^+_\phi$

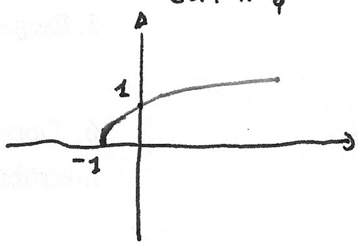


D: $\mathbb{R}$; Cd: $\mathbb{R}$



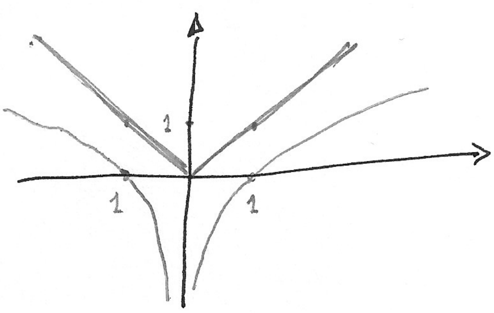
D: $\mathbb{R}^+_\phi$

Cd: $y \in [-1; +\infty]$



7) $P(4T/4 \text{ lanci}) = \frac{C_{4;4}}{2^4} = \frac{\frac{4!}{(4-4)! \cdot 4!}}{16} = \frac{1}{16} \cdot \frac{\cancel{4!}}{\phi! \cdot \cancel{4!}} = \frac{1}{16} \cdot \frac{1}{1} = \underline{\frac{1}{16}}$

LODE)



$\underline{\ln|x| < |x| \quad \forall x \in \mathbb{R} \setminus \{\phi\}}$